5. R. I. Nigmatulin, "Equations of hydrodynamics and compression waves in a two-velocity and two-temperature continuous medium in the presence of phase transitions," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1967).
6. R. I. Nigmatulin, "Fine-scale flows and surface effects in the hydrodynamics of multiphase media," Prikl. Mat. Mekh., 35, No. 3 (1971).
7. G. A. Davidson, "A Burgers' equation approach to finite amplitude acoustics in aerosol media," J. Sound Vibr., 38, No. 4 (1975).
8. G. A. Davidson, "A Burgers' equation for finite amplitude acoustics in fogs," J. Sound Vibr., 45, No. 4 (1976).
9. F. E. Marble, "Dynamics of dusty gases," Ann. Rev. Fluid Mech., 2, 397 (1970).
10. A. L. Polyakova, S. I. Soluyan, and R. V. Khokhlov, "On the propagation of finite disturbances in a relaxing medium," Akoust. Zh., 8, No. 1 (1962).
11. G. B. Whitham, Linear and Nonlinear Waves, Wiley, New York (1974).
12. V. E. Nakoryakov and A. A. Borisov, "Propagation of disturbances in a medium with relaxation and a chemical reaction," Fiz. Goreniya Vzryva, 12, No. 3 (1976).
13. A. A. Borisov, A. F. Vakhgel't, and V. E. Nakoryakov, "Propagation of long-wavelength disturbances of finite amplitude in gas suspensions," Zh. Prikl. Makh. Tekh. Fiz., No. 5 (1980).

VORTICAL SINGULARITY OF SELF-SIMILAR GAS FLOW IN THE REFLECTION OF A SHOCK WAVE

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We consider the problem of the transient gas flow developing when a plane shock wave, propagating through a stationary homogeneous gas 0 and characterized by a Mach number $M = u/a_0$, strikes a rigid impermeable wall forming a wedge with an angle θ at the time $t = 0$. We consider the case in which the angle θ is large enough that the wave configuration represented in Fig. 1 developed in the interaction, where CD is the incident wave, BC is the regularly reflected wave, BE is the front of the rarefaction wave propagating from AO through the region of uniform flow 2, AB is the diffracted wave, and 3 is the region of nonuniform flow. In the absence of a characteristic size the problem is self-similar with arguments $x = X/t$ and $y = Y/t$.

In the case of $\theta \to \pi/2$ [1, 2], $u = v = 0$ at point O, i.e., the flow is stagnant. An asymptotic analysis as $\theta \to 0$ indicates that in the vicinity of point O flow takes place around the corner with conservation of most of the X component of the velocity. One can assume that in the intermediate case of $\theta < \pi/2$ flow around the corner takes place near point O and gas particles which have entered 3 through point A on the diffracted shock wave are located at the wall OE (at some part of it) adjacent to point O. On the other hand, obviously, particles which have entered 3 from 2 under the action of the rarefaction wave BE are located on the part of the wall OE adjacent to point E. The point separating the gas which has reached OE in different ways is designated as F in Fig. 1.

If one assumes that the gas is nonviscous and thermally nonconducting, then one can state that the entropy per particle is conserved during motion inside 3, and therefore particles on different sides of point F have different entropies and, because of the continuity of pressure, different densities. Point F, a vortical flow singularity [3, 4] of the type under consideration, corresponds to a Furry singularity of conical gas flows [5].

The vortical singularity (VS) at point F is an essential element of the fundamental problem of reflection of a shock wave from a wedge under consideration. In each of the approaches (analytical, numerical, or experimental) the motifs connected with its existence must be taken into account. Very satisfactory results obtained using a variant of a numeri-

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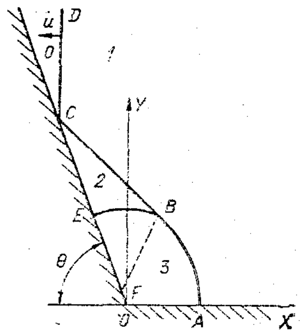

Fig. 1

cal scheme with isolation of the shock, in the construction of which the presence of a vortical singularity was taken into account, to calculate the flow in 3 are reported in [3], for example.

From the foregoing it follows that a particle which was at point O at the moment of arrival of the incident shock wave at it is located at F, i.e., point F is the origin of self-similarity in the Lagrangian coordinates. We can show that all the particle trajectories in the self-similar flow in 3 converge at point F of the xy plane.

Let us consider two times t_1 and t_2, $t_1 \ll t_2$ and $t_1 > 0$. At the scale of t_2, region 3, as it was at the time t_1 in the XY plane, can be taken as the vicinity of point O, while the time t_1 itself can be taken as close to zero. From the fact that F is the Lagrangian coordinate of the particle which was at point O at the time t = 0 it follows that by the time t_2 the particles distributed over region 3 at the time t_1 are drawn together toward point F (at the scale t_2). The interpretation of the corresponding theorem from the theory of conical flows [5] in the case of flow under consideration can be made the formal basis of this conclusion.

In the XY plane at the time t_1 we introduce a certain closed loop $L(t_1)$ lying entirely inside 3. Gas particles with a mass m are located in the part of region 3 with an area $\Sigma(t_1)$ bounded by L. The motion of these particles in the space XYt takes place inside a certain stream tube with a cross section $\Sigma(t)$. We designate the transform of L(t) in the xy plane as $l(t)$. Evidently, the area $\sigma(t)$ of the figure bounded by l is $\sigma(t) = (1/t^2)\Sigma(t)$. Then we introduce into the analysis ρ_e, the lower estimate for ρ_3 ($\rho_3 > \rho_e$), and then $\Sigma(t) = m/\rho_{3av} < m/\rho_e = \text{const} = \Sigma^*$.

This means that $(1/t^2)\Sigma^*$ is an upper estimate for $\sigma(t)$. Consequently, the fluid loop $l(t)$ in xy contracts with an increase in t, while the streamlines in xy converge to singular points or a profile. If we repeat the above for a loop L which partially coincides with the wall, then we find that any fluid loop contracts toward the wall. Hence a singularity is located at the wall. It was found above that two gas particles which were adjacent near point F entered 3 by traveling the paths AOF and EF (in the xy plane), respectively. Therefore, streamlines of the nonuniform flow 3 in the xy plane converge toward a singular point which lies at the wall, coincides with F, and is unique.

From the smoothness of the shock wave ABC it follows that this point is a singularity of the "node" type for density and entropy. It is appropriate to note that a smooth reflected shock wave does not generate a contact discontinuity at point B, while at BF, the line separating the gas which passed through AB and BC, the density is continuous down to F.

The fundamental possibility of analyzing the flow in the vicinity of a VS is connected with the use of a solution of the problem for flow in 3 which is linear with respect to $\delta = \pi/2 - \theta$ [2]. In this solution an equation is obtained for $\rho = (\rho_3 - \rho_2)/\rho_2\delta$ ($\varkappa = 7/5$, the designations of the parameters are standard, and the index corresponds to the number of the zone),

$$\rho(x, y) = \begin{cases} p(x, y) + (C_0 - 1)\, p\left(k, k\frac{y}{x}\right), & \frac{y}{x} < \tan \measuredangle AOB, \\ p(x, y), & \frac{y}{x} \geqslant \tan \measuredangle AOB, \end{cases} \tag{1}$$

where

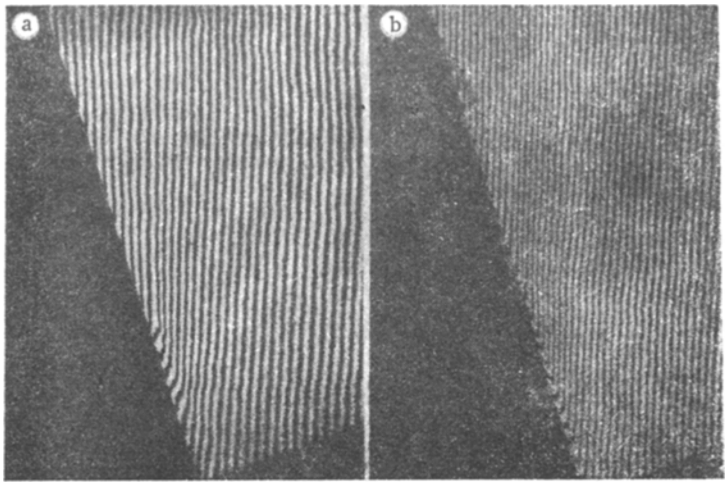

Fig. 2

$$p(x, y) = \sum_{n=0}^{\infty} \frac{1}{2} C_{2n+1} \left[\left(\frac{1+x}{1-x}\right)^{\frac{2n+1}{2}} + \left(\frac{1-x}{1+x}\right)^{\frac{2n+1}{2}} \right] \sin\left\{(2n+1)\frac{\pi}{2} + (2n+1)\,\mathrm{arctg}\,\frac{y}{\sqrt{1-x^2-y^2}}\right\};$$

$$C_0 = \frac{3}{5}\sqrt{\frac{4M^2-1}{M^2+2}}\,\frac{(M^2+5)(4M^2-1)}{(7M^2-1)(M^2+2)};\quad k = \sqrt{\frac{M^2+2}{4M^2-1}}\ (k = x_A,\ \delta = 0)$$

(the remaining C_i are coefficients defined in [2]). Several important conclusions follow from Eq. (1). First, point F is located in xy at a distance of $\sim\delta$ from the point 0, i.e., the velocity of a particle coinciding with F has the order δ. Second, the variation of ρ from ray to ray in the sector BFE is small ($o(\delta)$) and is important only in the sector BFO, where it has the order δ. Finally, most of the increment of density in the fan from the ray FO to the ray FC is

$$[\rho_3]_F \approx (C_0 - 1)\,\frac{p_{2A} - p_2}{\varkappa p_2}\,\rho_2. \tag{2}$$

These results open up the most interesting properties of VS for the most part. The difficulty of experimental observation of VS is primarily connected with the local nature of the phenomenon and the small size of the inhomogeneity. We can also note that the action of transfer effects inherent to a real gas smears out the picture and makes the experiment difficult. There are no data in the literature [3, 4, 6-8] on the experimental observation of VS.

All the same, having suitably selected the experimental regime, one can clearly observe the flow pattern in the vicinity of a vortical singularity. In this case one must strive to use a method with high spatial resolution and avoid submerging the VS in the developed boundary layer at the wall, which, although it does not prevent recording the VS, still distorts the picture somewhat.

To observe a vortical singularity in an air shock tube (without evacuation) with a rectangular cross setion of 80 × 130 mm we used a diffraction interferometer assembled on a shadow instrument operating with a full shift of the wavefronts. In this way it is converted into an interferometer of the Mach–Zehnder type. Photographs were made on aerial photography film 320 mm wide, and the number of bands in the field reached 200. To improve the spatial resolution it is desirable to adjust the interferometer for the closest possible bands.

In Fig. 2a-b we give characteristic results obtained for a wedge with $\delta = 20°$ and $M = 1.772$ and 1.796, respectively. In the photographs a section of the wall coincides with the lower edge. A magnified fragment of the field in the vicinity of points O and F is given, since in printing it is difficult to clearly reproduce an entire interferogram in such close bands. The fronts of waves ABC and BE lie outside the frame. We can note that a theoretical estimate of the thickness of the transient boundary layer in the vicinity of a VS leads to a value of $\sim$0.1-0.5 band width.

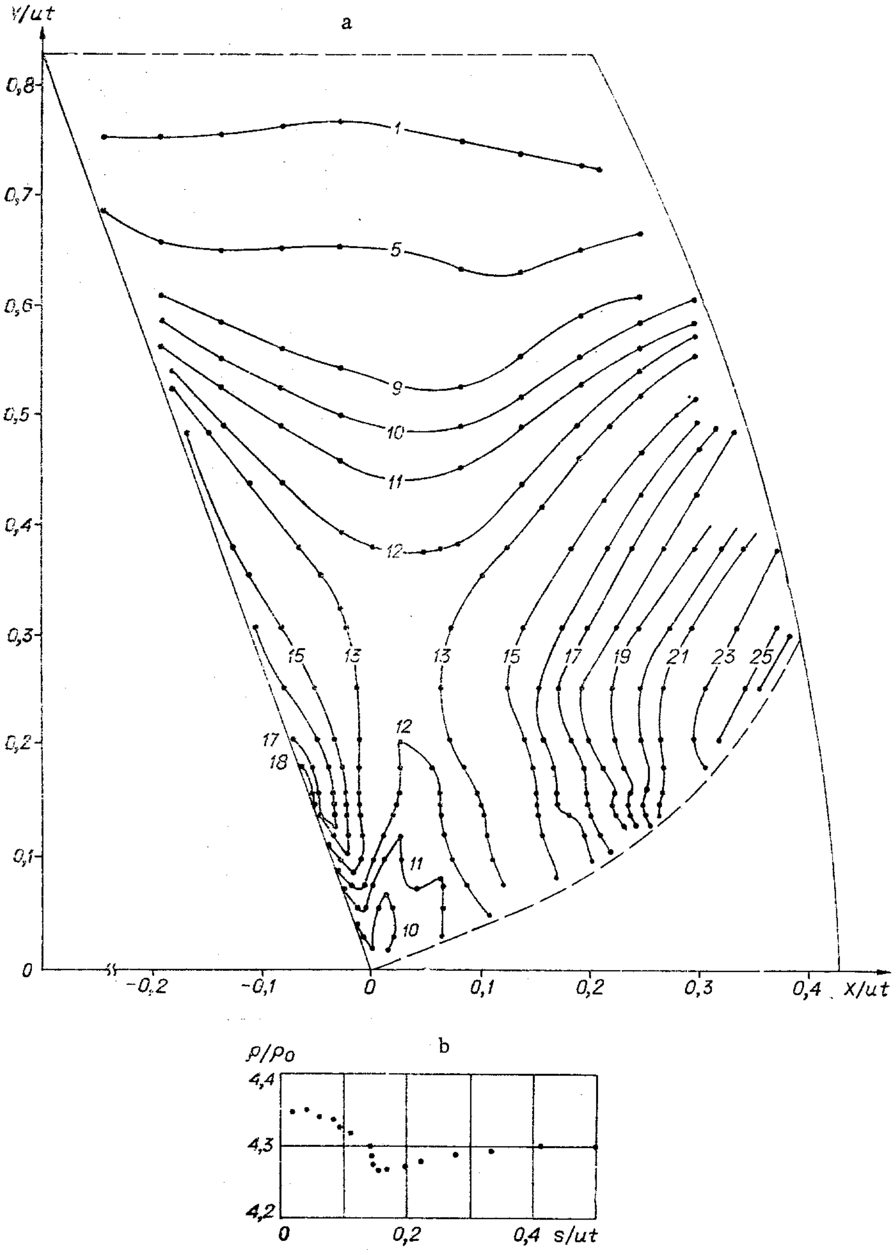

Fig. 3

A vortical singularity is revealed by the characteristic behavior of the bands in its vicinity, and the coordinates of the VS can be determined with an accuracy of several percent. This is a unique possibility for the direct determination of the gas velocity at a characteristic point of the flow.

The density distribution in the flow field can be obtained from an interferogram. The result corresponding to Fig. 2a is given in Fig. 3a. Lines of constant density converge in a fan at a singular point of the "node" type. A comparison of the results of the interpretation and Fig. 2a increases the accuracy in determining the coordinates of the VS. The density distribution right along the wall is given in Fig. 3b (s is the coordinate measured along the wall from the vertex O).

Equation (2) gives a value of $0.052\rho_0$ for the discontinuity in density at point F in this case. This result is very close to the value represented in Fig. 3b, despite the fact that the angle δ is not particularly small in the given case. As was mentioned, the density distribution differs from an ideal one because of the presence of transfer processes. The lengths in Fig. 3 are normalized to the distances from the vertex to the incident wave (ut).

As to the interferogram, it is desirable to fix the source function of quantitative information about the VS, and it is most convenient to observe this phenomenon on a color shadow diagram. The eye distinguishes color gradations more effectively than gradations of gray or microscopic deviations of lines on an interferogram. We obtained color shadow diagrams from interferograms with the closest bands [9], which act as holograms in this case. In our experiments we first saw the VS on the color shadow diagrams and then established that the singular point moves along the surface of the wedge from the vertex of the corner with a constant velocity. In a color picture the VS shows up in the form of a contrasting color spot. In it one can also observe a band moving from the vortical singularity toward the point of encounter of the regularly reflected shock wave and the bow rarefaction wave (point B).

From the analysis given above it follows that contact between the gas which entered 3 through AB and the gas which passed through BE takes place in the region of the band. Since BE is a rarefaction wave, there is no density discontinuity, and we can assume that the method reveals a nonuniform distribution of the density gradient in the region of the "contact surface" BF. A contrasting formation, a "contact surface," emerges in a similar way from the point of encounter of incident and diffracted shock waves and of the rarefaction wave in shadow photographs of the flow field arising in the diffraction of a shock wave near a flat corner [6, 7]. As in 3, at this surface there is no density discontinuity, in contrast to the contact surface of their ensemble of a three-shock configuration.

In the diffraction of a shock wave near a flat corner there is also a vortical singularity, fully analogous to that discussed above. However, the conditions of the experiment with a VS in the reflection problem are more favorable, since in this case the gas velocity relative to the wall in the vicinity of the singular point is low and transfer effects do not deviate the phenomenon too much from the ideal model.

LITERATURE CITED

1. M. J. Lighthill, "The diffraction of blast. II," Proc. R. Soc. London, Ser. A, 200, 554 (1950).
2. E. F. Zhigalko, "Nearly normal reflection of a shock wave in a linear approximation," Vestn. Leningr. Gos. Univ., No. 13 (1970).
3. P. Kutler and V. S. Vijaya Shankar, "Diffraction of a shock wave by a compression corner. 1. Regular reflection," AIAA J., 15, 197 (1977).
4. G. Ben Dor and I. I. Glass, "Nonstationary oblique shock wave reflections: Actual isopycnics and numerical experiments," AIAA J., 16, No. 11 (1978).
5. B. M. Bulakh, Nonlinear Conical Gas Flows [in Russian], Nauka, Moscow (1970).
6. T. V. Bazhenova and L. G. Gvozdeva, Transient Interactions of Shock Waves [in Russian], Nauka, Moscow (1970).
7. B. W. Skews, "The shape of diffracting shock wave," J. Fluid Mech., 29, Pt. 2 (1967).
8. C. K. Law and I. I. Glass, "Diffraction of strong shock waves by a sharp compressive corner," Can. Aeronaut. Space Inst. Trans., 4, 99 (1971).
9. L. L. Kolyshkina and E. F. Zhigalko, "Analysis of a color shadow picture obtained from a hologram," Zh. Tekh. Fiz., No. 2 (1981).